Together these cover all the four-valuation truth tables – either with just a sentence letter (as in tables 8 and 9) or with a single connective. Here's a 'group portrait', the culmination of our connective spending spree.

(P → P)	(P → ~Q)	(P → Q)	(Q → P)	(~P → Q)	((P → ~Q) → ~(~Q → P))	(P → ~P)	(~P → P)	(~Q → Q)	(Q → ~Q)	((P → Q) → ~(Q → P))	~(P → ~Q)	~(P → Q)	~(Q → P)	~(~Q → P)	~(P → P)
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
1	0	1	1	1	1	0	1	1	0	0	1	0	0	0	0
1	1	0	1	1	0	0	1	0	1	1	0	1	0	0	0
1	1	1	0	1	0	1	0	1	0	1	0	0	1	0	0
1	1	1	1	0	1	1	0	0	1	0	0	0	0	1	0

And here are all the **expressively adequate, non-redundant languages** built from these connectives.

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$(P \to P)$	$(P \to \sim Q)$	$(P \to Q)$	$(Q \to P)$	$(\sim P \to Q)$	$((P \to \sim Q) \to \sim(\sim Q \to P))$	$(P \to \sim P)$	$(\sim P \to P)$	$(\sim Q \to Q)$	$(Q \to \sim Q)$	$((P \to Q) \to \sim(Q \to P))$	$((P \to \sim Q) \to \sim(P \to \sim Q))$	$((P \to Q) \to \sim(P \to Q))$	$((Q \to P) \to \sim(Q \to P))$	$((\sim Q \to P) \to \sim(\sim Q \to P))$	$((P \to P) \to \sim(P \to P))$
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
1	0	1	1	1	1	0	1	1	0	0	1	0	0	0	0
1	1	0	1	1	0	0	1	0	1	1	0	1	0	0	0
1	1	1	0	1	0	1	0	1	0	1	0	0	1	0	0
1	1	1	1	0	1	1	0	0	1	0	0	0	0	1	0

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Together these cover all the four-valuation truth tables – either with just a sentence letter (as in tables 8 and 9) or with a single connective. Here’s a ‘group portrait’, the culmination of our connective spending spree.

$(P \to P)$	$(P \to (Q \to \bot))$	$(P \to Q)$	$(Q \to P)$	$((P \to \bot) \to Q)$	$(((P \to Q) \to ((Q \to P) \to \bot)) \to \bot)$	$(P \to \bot)$	$((P \to \bot) \to \bot)$	$((Q \to \bot) \to \bot)$	$(Q \to \bot)$	$((P \to Q) \to ((Q \to P) \to \bot))$	$((P \to (Q \to \bot)) \to \bot)$	$((P \to Q) \to \bot)$	$((Q \to P) \to \bot)$	$(((Q \to \bot) \to P) \to \bot)$	$((P \to P) \to \bot)$
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
1	0	1	1	1	1	0	1	1	0	0	1	0	0	0	0
1	1	0	1	1	0	0	1	0	1	1	0	1	0	0	0
1	1	1	0	1	0	1	0	1	0	1	0	0	1	0	0
1	1	1	1	0	1	1	0	0	1	0	0	0	0	1	0

And here are all the **expressively adequate, non-redundant languages** built from these connectives.

Equivalent:

$$((P \to (Q \to \bot)) \to \bot) \;= (\,(P \to ((P \to Q) \to \bot)) \to \bot\,) = (P \land Q)$$

$$((P \to (Q \to \bot)) \to (((Q \to \bot) \to P) \to \bot))$$

$$= ((P \to Q) \to ((Q \to P) \to \bot))$$

$$= (P \leftrightarrow Q)$$

3.16.19:

$$((P \lor Q) \to (P \land Q)) \;\approx\; (P \leftrightarrow Q) \;\approx$$

$$(\;((P \to \bot) \to Q) \;\to\; ((P \to (Q \to \bot)) \to \bot\,)\;)$$

Indirect Deduction with $\{\rightarrow, \perp\}$:

To show P, assume $(P \rightarrow \perp)$
Box closes when: get some sentence, X, and also $(X \rightarrow \perp)$

(NB: from X and $(X \rightarrow \perp)$, get $\perp$ by MP. So just have box close when we get $\perp$.)

Vel Elim	1. $(P \vee Q)$ 2. $\sim P$ $\therefore$ Q	1. $((P \rightarrow \perp) \rightarrow Q)$ 2. $(P \rightarrow \perp)$ $\therefore$ Q
Vel Intro	1. P $\therefore$ $(P \vee Q)$	1. P $\therefore$ $((P \rightarrow \perp) \rightarrow Q)$
Wedge Elim	1. $(P \wedge Q)$ $\therefore$ P	1. $((P \rightarrow (Q \rightarrow \perp)) \rightarrow \perp)$ $\therefore$ P
Wedge Elim	1. $(P \wedge Q)$ $\therefore$ Q	1. $((P \rightarrow (Q \rightarrow \perp)) \rightarrow \perp)$ $\therefore$ Q
Wedge Intro	1. P 2. Q $\therefore$ $(P \wedge Q)$	1. P 2. Q $\therefore$ $((P \rightarrow (Q \rightarrow \perp)) \rightarrow \perp)$
Tilde Elim	1. $\sim\sim P$ $\therefore$ P	1. $((P \rightarrow \perp) \rightarrow \perp)$ $\therefore$ P
Tilde Intro	1. P $\therefore$ $\sim\sim P$	1. P $\therefore$ $((P \rightarrow \perp) \rightarrow \perp)$
Modus Tollens	1. $(P \rightarrow Q)$ 2. $\sim Q$ $\therefore$ $\sim P$	1. $(P \rightarrow Q)$ 2. $(Q \rightarrow \perp)$ $\therefore$ $(P \rightarrow \perp)$
1.	1.	1.

Rules of Inference:

Modus Ponens (MP)

$(\bullet \rightarrow \blacktriangle)$

$\bullet$

$\blacktriangle$

Transitivity (T)

$(\bullet \rightarrow \blacktriangle)$

$(\blacktriangle \rightarrow \clubsuit)$

$(\bullet \rightarrow \clubsuit)$

Indirect Deduction

~~Get~~: $\bullet$ (ID)

$(\bullet \rightarrow \perp)$ AID

.

.

.

$\perp$

$\bullet$ 2, 7, 8, ID

Conditional Deduction

~~Get~~: $(\bullet \rightarrow \blacktriangle)$ (CD)

$\bullet$ ACD

.

.

.

$\blacktriangle$

$(\bullet \rightarrow \blacktriangle)$ x, y, CD

Can get T from CD, MP